Analysis of Mode Coupling on Guided-Wave Structures Using Morse Critical Points

Alexander B. Yakovlev, *Member, IEEE,* and George W. Hanson, *Member, IEEE*

***Abstract*— New insight on mode coupling in waveguiding structures is obtained from the theory of Morse critical points (MCP's). It is shown that the traditional coupled-mode formalism has a clear analytical connection with functional properties of the characteristic determinant in the vicinity of the Morse critical point, which determines the minimum of coupling. The relationship between perturbed and independent modes in the mode-coupling region is obtained using the Taylor polynomial of order two about the Morse critical point, and it is found that the coupling factor is proportionally related to the value of a characteristic function at this point. The qualitative modal behavior in the mode-interaction region is predicted by a simple normal form, which can be geometrically interpreted as a result of the intersection of a saddle surface and a plane corresponding to the minimum of the coupling factor. Numerical results for a variety of guided-wave structures, including printed-circuit transmission lines, planar-slab waveguides, and shielded microstrip-like lines demonstrate the efficiency of the proposed approach for the rapid identification of mode-coupling regions, and for reconstruction of dispersion behavior in those regions via simple analytic (normal) forms.**

***Index Terms*—Coupling, coupled-mode analysis, critical points, electromagnetic coupling, transmission lines, waveguides.**

I. Introduction

MORE than 40 years ago Pierce, investigating the coupling of modes in microwave traveling-wave tubes [1], formulated the relationship between modes before and after coupling, and introduced the concept of a coupling factor between waveguide modes. Later on, a great number of papers appeared in the scientific literature showing development of the coupled-mode theory and applications for microwave and optical waveguides [2]–[4] and devices, including directional couplers [5], periodic structures [6], nonparallel waveguiding structures [7], grating couplers [8], etc.

Various mathematical approaches have been applied for the analysis of coupled waveguiding structures. A general review of coupled-mode theory is provided in [9], as well as providing some examples of coupling in resonator and waveguiding systems via a variational principle. The application of a generalized reciprocity theorem has been proposed for mode-coupling study in parallel dielectric waveguiding structures [10]. It is shown that results obtained by the use of a generalized reciprocity relation and by a variational principle are identical. A coupled-mode theory for multilayered and multiconductor transmission lines has been developed using a generalized reciprocity theorem as well [11]. The coupling coefficients are obtained in terms of the overlap integrals connecting eigenmode fields and currents of individual conductors. Full-wave perturbation theory has been applied for the accurate analysis of coupled microstrip transmission lines [12] and resonant structures [13]. The numerical results agree well with those obtained by a Galerkin's moment-method solution and with experimental data.

A different view on mode coupling is based on the principles of catastrophe theory [14]–[16]. Analysis of the eigenmode mutual coupling using the concept of Morse critical points (MCP's) from catastrophe theory has been originally introduced for open resonators and open waveguides [17]. It has been observed that two solutions form a coupling diagram in the vicinity of the MCP found in the mode-coupling region. An effect of resonant and intertype oscillations in open two- and three-dimensional resonators has been investigated using the rigorous spectral theory of open structures [18] in conjunction with the concept of MCP's [19]. This idea has been also applied for the analysis of the eigenmode and induced-mode coupling in open waveguide resonators [20]. Coupling effects of complex waves have been considered in multilayer cylindrical strip and slot lines [21]. It was found that isolated MCP's and degeneration points have clear physical connection with observed dispersion behavior. Some results on free oscillations and intertype waves in diffraction gratings and their explanation by means of the analysis of singular points have been collected in [18]. A "nonphysical" complex regime of spectral curves has been found, which will be explained later in this paper, based on the analysis of the normal form defined by the MCP.

In this paper, the connection with the traditional coupled-mode formalism is demonstrated in terms of structural characteristics in the vicinity of MCP's. It is shown that simple analytical expressions for perturbed and independent modes in the mode-coupling regions can be obtained from the Taylor polynomial of order two about the MCP. Qualitative behavior of spectral curves in the region of interest is predicted by the analytically constructed normal form. A geometrical interpretation of the normal form is given as a result of the intersection of a saddle surface and a plane corresponding to the minimum of coupling. A clear explanation of the dispersion curves' hyperbolic behavior, and conditions for the occurrence of complex mode regimes is shown. The relationship between MCP's and fold points associated with leakage [22] is demonstrated for the case when a complex mode occurs in the mode-coupling region. Open and shielded

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The authors are with the Department of Electrical Engineering and Computer Science, University of Wisconsin–Milwaukee, Milwaukee, WI 53211 USA.

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guided-wave structures are investigated to show the generality of the obtained results.

II. Theory

Consider a mathematical model of a longitudinally invariant guided-wave structure resulting in a homogeneous matrix system for natural modes of the structure

$$A(\kappa, f)X = 0 \tag{1}$$

where $A(\kappa, f)$ is an operator-function, κ and f represent the unknown guided-wave normalized propagation constant k_g/k_0 and frequency, respectively, and X would typically represent a vector of unknown current density or electric field intensity, depending on a particular problem formulation. Imposing the necessary condition $\det[A(\kappa, f)] = 0$ leads to the determination of the propagation constant spectrum, which can include regular and singular (critical) points associated with certain modal behavior [22]. Functional characteristics in the local neighborhood of a regular point can be analyzed using the implicit function theorem [14], where a unique curve $\kappa = \kappa(f)$ or $f = f(\kappa)$ through a regular point can be obtained. To investigate the structural behavior at singular points, the smooth analytical function $H(\kappa, f) = \det[A(\kappa, f)]$ has been considered in the complex domain (κ, f).

It has been shown, for the explicit example of a conductor-backed coplanar strip line [22], that the MCP generally defined by the following set of equations:

$$\begin{aligned} H'_\kappa(\kappa, f)|_{(\kappa_m, f_m)} &= H'_f(\kappa, f)|_{(\kappa_m, f_m)} = 0 \\ \xi &= [H''_{\kappa\kappa}H''_{ff} - H''_{\kappa f}H''_{f\kappa}]|_{(\kappa_m, f_m)} \neq 0 \end{aligned} \tag{2}$$

exists in the mode-coupling region when two characteristic curves approach each other and bend before crossing. The universal mode-coupling behavior observed in a variety of guided-wave structures is related to the case when the Hessian determinant ξ is negative. The structural behavior of $H(\kappa, f)$ in the vicinity of the MCP (κ_m, f_m) can be analyzed using a Taylor series expansion, and the qualitative and quantitative local structure reconstructed. In the case when the Hessian determinant ξ is positive, the MCP defines local minima and maxima [14], [15]. This case is not of interest in this research and will not be considered here. Note that for the rest of this paper, the functional characteristics in the local neighborhood of the MCP ($H(\kappa_m, f_m)$, $H''_{\kappa\kappa}(\kappa_m, f_m)$, $H''_{ff}(\kappa_m, f_m)$, $H''_{\kappa f}(\kappa_m, f_m)$, $H''_{f\kappa}(\kappa_m, f_m)$) are obtained as real-valued quantities. This will be the case when the MCP (κ_m, f_m) is real valued, which occurs for lossless media even in the event of complex modes. The MCP formalism, i.e., (2), (4), etc., is equally valid at complex points (κ_m, f_m), which occur for lossy media, although, in this case, the connection with traditional coupled-mode theory is not as straightforward. Therefore, in the following, lossless structures will be considered.

To obtain a general relationship between the concept of MCP's and the traditional coupled-mode formalism, we use the result of the Morse lemma [14], [16], which proves that the function H in the local neighborhood of the MCP can be exactly represented by a quadratic canonical form using a smooth change of coordinates. Therefore, it is enough to consider the Taylor polynomial of order two about the MCP

$$\begin{aligned} H(\kappa, f) = {} & H(\kappa_m, f_m) + \tfrac{1}{2} H''_{\kappa\kappa}(\kappa - \kappa_m)^2 \\ & + H''_{\kappa f}(\kappa - \kappa_m)(f - f_m) + \tfrac{1}{2} H''_{ff}(f - f_m)^2 \end{aligned} \tag{3}$$

where all partial derivatives are calculated at (κ_m, f_m) and $H''_{\kappa f} = H''_{f\kappa}$. Local structure $\kappa = \kappa(f)$ can be easily obtained from the quadratic form (3), shown in (4), at the bottom of this page.

$$\kappa_{1,2} = \kappa_m - \frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f - f_m) \pm \frac{\sqrt{((H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff})(f - f_m)^2 - 2H''_{\kappa\kappa}H(\kappa_m, f_m)}}{H''_{\kappa\kappa}} \tag{4}$$

The expression (4) represents the local behavior of the propagation constants $\kappa_{1,2} = \kappa_{1,2}(f)$ in the vicinity of the MCP (κ_m, f_m), which is located in the mode-coupling region of a perturbed waveguiding structure. It has been discussed [22] that the point of modal degeneracy $H(\kappa_m, f_m) = 0$ is related to a double-point bifurcation, and the solution is represented as two intersecting straight lines. The condition $H(\kappa_m, f_m) = 0$ can be associated with unperturbed modes [23] with the propagation constants $\tilde{\kappa}_{1,2}$ given by

$$\tilde{\kappa}_{1,2} = \kappa_m + S_{1,2}(f - f_m) \tag{5}$$

where $S_{1,2}$ represent slopes of intersecting straight lines

$$S_{1,2} = -\frac{H''_{\kappa f}}{H''_{\kappa\kappa}} \pm \sqrt{\left(\frac{H''_{\kappa f}}{H''_{\kappa\kappa}}\right)^2 - \frac{H''_{ff}}{H''_{\kappa\kappa}}}. \tag{6}$$

It can be observed that the definition of codirectional forward (backward) and contradirectional waves is connected with functional characteristics in the vicinity of the MCP expressed in terms of $S_{1,2}$ defined by (6) as follows:

Codirectional Forward Waves:

$$S_{1,2} > 0 \Longrightarrow \frac{H''_{\kappa f}}{H''_{\kappa\kappa}} < 0 \quad \frac{H''_{ff}}{H''_{\kappa\kappa}} > 0 \tag{7}$$

Codirectional Backward Waves:

$$S_{1,2} < 0 \Longrightarrow \frac{H''_{\kappa f}}{H''_{\kappa\kappa}} > 0 \quad \frac{H''_{ff}}{H''_{\kappa\kappa}} > 0 \tag{8}$$

Contradirectional Waves:

$$\left\{\begin{matrix} S_1 > 0 \\ S_2 < 0 \end{matrix}\right\} \Longrightarrow \frac{H''_{\kappa f}}{H''_{\kappa\kappa}} > 0 \quad \frac{H''_{ff}}{H''_{\kappa\kappa}} < 0. \tag{9}$$

Coordinates of the MCP (κ_m, f_m) can be expressed in terms of unperturbed modes' propagation constants $\tilde{\kappa}_{1,2}$ determined by (5) and (6)

$$\begin{aligned} \kappa_m &= \frac{(\tilde{\kappa}_1 + \tilde{\kappa}_2)}{2} + \frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f - f_m) \\ f_m &= f - \frac{(\tilde{\kappa}_1 - \tilde{\kappa}_2)}{2} \frac{H''_{\kappa\kappa}}{\sqrt{((H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff})}}. \end{aligned} \tag{10}$$

Substitution of (κ_m, f_m) given by (10) into (4) leads to the relationship between $\kappa_{1,2}$ and $\tilde{\kappa}_{1,2}$

$$\kappa_{1,2} = \frac{(\tilde{\kappa}_1 + \tilde{\kappa}_2)}{2} \pm \sqrt{\left(\frac{\tilde{\kappa}_1 - \tilde{\kappa}_2}{2}\right)^2 - 2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}}. \tag{11}$$

In contrast to (11), the following result has been obtained for mode coupling in microwave traveling-wave tubes [1]:

$$\kappa_{1,2} = \frac{(\tilde{\kappa}_1 + \tilde{\kappa}_2)}{2} \pm \sqrt{\left(\frac{\tilde{\kappa}_1 - \tilde{\kappa}_2}{2}\right)^2 \pm \left(\frac{K}{k_0}\right)^2} \tag{12}$$

which expresses the result of coupling of independent (unperturbed) modes $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$, and K/k_0 represents the normalized coupling factor between modes $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$. Signs "±" at K/k_0 term are related to modes with codirectional and contradirectional power flow, respectively.

Comparison of (11) and (12) makes it possible to conclude that the mode-coupling factor K/k_0 is connected with functional characteristics at the MCP (κ_m, f_m)

$$\pm\left(\frac{K}{k_0}\right)^2 = -2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}$$

which leads to the expression of K/k_0 for codirectional coupling

$$\frac{K}{k_0} = \sqrt{-2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}} \tag{13}$$

and contradirectional coupling

$$\frac{K}{k_0} = \sqrt{2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}}. \tag{14}$$

It should be noted that the case when $H(\kappa_m, f_m)/H''_{\kappa\kappa} < 0$ is related to codirectional power flow and mode coupling between two forward (backward) traveling waves. Occurrence of complex modes in the mode-coupling region between a forward wave and a backward wave is connected with the case when $H(\kappa_m, f_m)/H''_{\kappa\kappa} > 0$. The results (13) and (14) represent the connection between coupled mode theory and the concept of MCP's from catastrophe theory.

It has also been shown [23] that $\Delta\kappa = \kappa_1 - \kappa_2$ has the minimum value of the mode-coupling factor of $2K/k_0$ at the frequency which corresponds to a degeneracy of independent modes $\tilde{\kappa}_1$ and $\tilde{\kappa}_2$, leading to $\tilde{\kappa}_1 = \tilde{\kappa}_2$. The same result can be obtained considering $\kappa_{1,2}$ and $\tilde{\kappa}_{1,2}$ at $f = f_m$. Expression (5) explicitly shows that at frequency f_m the independent modes $\tilde{\kappa}_{1,2}$ are degenerated and $\tilde{\kappa}_1 = \tilde{\kappa}_2 = \kappa_m$. Using this result, the relationship (11) is reduced to

$$\kappa_{1,2} = \kappa_m \pm \sqrt{-2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}}.$$

As a result,

$$\Delta\kappa = \kappa_1 - \kappa_2 = 2\sqrt{-2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}}$$

which is consistent with the above expression (13) for the mode-coupling factor of codirectional waves. The result (14) for contradirectional coupling can be obtained from (11) considering imaginary parts of the propagation constants in the complex mode regime. It should be noted that the Morse frequency f_m is an important parameter for evaluating the minimum of the mode-coupling factor.

Using a similar procedure, the following relationship between $\tilde{\kappa}_{1,2}$ and $\kappa_{1,2}$ can be obtained:

$$\tilde{\kappa}_{1,2} = \frac{(\kappa_1 + \kappa_2)}{2} \pm \sqrt{\left(\frac{\kappa_1 - \kappa_2}{2}\right)^2 + 2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}} \tag{15}$$

and compared with the result given in [23, eqs. (7) and (8)].

In this section of the paper, the connection between the concept of MCP's and the coupled-mode theory is shown via simple analytical relations (11)–(15). It is observed that the Morse frequency f_m determines the minimum of coupling and can be effectively used for the identification of mode-coupling regions.

Before proceeding to examine the specific cases of co- and contradirectional coupling, it is worthwhile to illustrate the ideas presented above from a slightly different perspective. Rather than defining the Morse point as in (2), and considering the degenerate condition $H = 0$ (double-point bifurcation) as a special case at which point unperturbed modes exist (which was done in the proceeding), assume initially that the structure in question exhibits sufficient symmetry such that two classes of modes (e.g., even and odd) of propagation will exist independently of each other. Let $H(\kappa, f) = 0$ be the dispersion equation which implicitly determines $\kappa(f)$. The conditions [24]

$$\begin{aligned} H(\kappa_m, f_m) &= H'_\kappa(\kappa, f)|_{(\kappa_m, f_m)} = H'_f(\kappa, f)|_{(\kappa_m, f_m)} = 0 \\ \xi &= [H''_{\kappa\kappa}H''_{ff} - H''_{\kappa f}H''_{f\kappa}]|_{(\kappa_m, f_m)} \neq 0 \end{aligned} \tag{16}$$

are necessary and sufficient for H to be locally equivalent around (κ_m, f_m) to the normal form

$$\gamma(\kappa^2 + \delta f^2) \tag{17}$$

where $\gamma = \mathrm{sgn}(H''_{\kappa\kappa})$, $\delta = \mathrm{sgn}(\xi)$, and $\mathrm{sgn}(x)$ denotes the sign of x, $x \neq 0$. As before, we are interested in the case when $\delta < 0$, such that (17) describes two intersecting straight lines. This is the situation when the dispersion curves of two uncoupled modes intersect, leading to a modal degeneracy. Now assume a small perturbation exists which breaks the original structural symmetry, resulting in coupling of the previously uncoupled modes. The dispersion equation may be written as

$$H(\kappa, f) + \epsilon p(\kappa, f, \epsilon) = 0 \tag{18}$$

where ϵ is some small parameter and p is a perturbation function. It can be shown [24] that a function $G(\kappa, f, \alpha)$ exists such that: 1) $G(\kappa, f, 0) = H(\kappa, f)$ and 2) for any small perturbation p there is an α such that $H+\epsilon p$ is equivalent to $G(\kappa, f, \alpha)$ (i.e., up to equivalence, G contains all small perturbations of H). Such a function G is called a universal unfolding of H, and for the degenerate Morse point defined by (16) the universal unfolding is

$$G(\kappa, f, \alpha)=\gamma(\kappa^2+\delta f^2+\alpha). \tag{19}$$

In general, a universal unfolding $G(\kappa, f, \alpha_1, \cdots, \alpha_k)$ of $H(\kappa, f)$ requires k additional parameters $\alpha_1, \cdots, \alpha_k$, where k is called the codimension of H. It is shown in [24] that the codimension of H satisfying (16) is unity and, thus, exactly one additional parameter α is needed to locally describe the effect of all possible small perturbations p on H. This is significant, since any such small perturbation (due to geometry, material inhomogeneity, etc.) is guaranteed to result in the behavior described by (19), which represents the characteristic hyperbolic form encountered in mode-coupling problems. Therefore, in this latter representation, behavior about the nondegenerate Morse point (with $H(\kappa_m, f_m)\neq 0$) associated with mode coupling (detailed previously) is obtained as a universal unfolding associated with the degenerate Morse point (with $H(\kappa_m, f_m)=0$) for the initial symmetrical structure. The universal unfolding (19) predicts the same qualitative form as the Taylor series (3). The definition of (2) together with the Morse lemma or (16) together with the concept of universal unfoldings lead to the same answer, although one method may be better suited for certain analyses that the other. For instance, the view of a perturbed symmetrical structure together with a universal unfolding makes it clear that, in a practical sense, the hyperbolic form (19) always occurs on physically realizable structures since any imperfection (perturbation), however small, due to structural/material asymmetry inherent in a manufacturing process, will result in mode coupling.

A. Mode Coupling Between Codirectional Waves

Mode coupling of codirectional waves is related to the case when $H(\kappa_m, f_m)/H''_{\kappa\kappa}<0$ as well as slope requirements (7) and (8) for forward (backward) waves expressed in terms of functional characteristics evaluated in the vicinity of the MCP. Local structure (4) in the case of mode coupling between codirectional waves is represented as a pure real valued function $\kappa(f)$, shown in (20), at the bottom of this page.

According to the definition of codirectional waves ($\kappa_{1,2}\, d\kappa_{1,2}/df > 0$ (<0) for forward (backward) waves), it can be shown using the representation (20) that slopes of $\kappa_{1,2}$ at f_m have the same sign

$$\left.\frac{d\,\mathrm{Re}\{\kappa_{1,2}\}}{df}\right|_{f=f_m}=-\frac{H''_{\kappa f}}{H''_{\kappa\kappa}}.$$

It can be observed that the function

$$\kappa(f)=\kappa_m-\frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f-f_m)$$

represents the equation of the straight line with the slope $-H''_{\kappa f}/H''_{\kappa\kappa}$. The linear coordinate transformation

$$\left(\kappa+\frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f-f_m),\, f\right)\Longrightarrow(\hat{\kappa}, f)$$

leads to the form shown in (20a), at the bottom of this page, which represents two branches of a hyperbola defined by the canonical form (normal form)

$$(\mathrm{Re}\{\hat{\kappa}-\kappa_m\})^2-\frac{((H''_{\kappa f})^2-H''_{\kappa\kappa}H''_{ff})}{(H''_{\kappa\kappa})^2}(f-f_m)^2=-2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}. \tag{21}$$

The result (21) shows the qualitative and quantitative behavior of dispersion curves in the mode-coupling region, which is geometrically approximated by two hyperbola branches centered at (κ_m, f_m), as will be demonstrated later. It should be noted that the function given in the left side of (21) represents the equation of a saddle surface in the three-dimensional space (κ, f, H). The expression in the right side corresponds to the equation of a plane related to the minimum of the coupling factor. As a result, the normal form (21) is geometrically represented as the intersection of a saddle surface and a plane, which results in hyperbolic behavior of local structure associated with mode coupling.

B. Mode Coupling Between Contradirectional Waves

Mode coupling between forward and backward traveling waves is related to the case when $H(\kappa_m, f_m)/H''_{\kappa\kappa}>0$ and slope requirements for contradirectional waves (9). The local structure (4) generated in the vicinity of the MCP represents real and complex solutions. If the following condition is satisfied in (4):

$$((H''_{\kappa f})^2-H''_{\kappa\kappa}H''_{ff})(f-f_m)^2-2H''_{\kappa\kappa}H(\kappa_m, f_m)<0 \tag{22}$$

$$\begin{aligned}\mathrm{Re}\{\kappa_{1,2}\}&=\kappa_m-\frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f-f_m)\pm\frac{\sqrt{((H''_{\kappa f})^2-H''_{\kappa\kappa}H''_{ff})(f-f_m)^2-2H''_{\kappa\kappa}H(\kappa_m, f_m)}}{H''_{\kappa\kappa}}\\ \mathrm{Im}\{\kappa_{1,2}\}&=0\end{aligned} \tag{20}$$

$$\mathrm{Re}\{\hat{\kappa}_{1,2}\}=\kappa_m\pm\frac{\sqrt{((H''_{\kappa f})^2-H''_{\kappa\kappa}H''_{ff})(f-f_m)^2-2H''_{\kappa\kappa}H(\kappa_m,f_m)}}{H''_{\kappa\kappa}} \tag{20a}$$

then local structure for the propagation constant of complex modes in the mode-coupling region is represented by a straight line with the slope $-H''_{\kappa f}/H''_{\kappa\kappa}$

$$\mathrm{Re}\{\kappa(f)\} = \kappa_m - \frac{H''_{\kappa f}}{H''_{\kappa\kappa}}(f - f_m). \tag{23}$$

The complex mode condition (22) allows determination of the frequency range of complex modes in terms of the Morse frequency and functional characteristics in the vicinity of the MCP

$$f \in \left(f_m - \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}},\; f_m + \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}\right) \tag{24}$$

and

$$\Delta f = 2\sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}.$$

If the complex mode condition (22) is not satisfied, the local structure (4) generates two real-valued branches of a hyperbola in opposite quadrants in comparison with the case of mode coupling of codirectional waves. The following frequency ranges correspond to hyperbolic behavior of propagation constants:

$$f < f_m - \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}$$

and

$$f > f_m + \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}.$$

To find the values of the propagation constants $\kappa_{1,2}$ which correspond to the boundary frequencies of the complex-mode regime in the mode-coupling region, we combine (23) and (24) and obtain the following values for $\kappa_{1,2}$:

$$\kappa_{1,2} = \kappa_m \mp \frac{H''_{\kappa f}}{H''_{\kappa\kappa}}\sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}. \tag{25}$$

The attenuation constant $\alpha = \mathrm{Im}\{\kappa_{1,2}\}$ can be expressed from local form (4) satisfying the frequency-range condition (24), shown in (26), at the bottom of this page. The maximum values of the attenuation constant α will be reached at $f = f_m$

$$\mathrm{Im}\{\kappa_{1,2}^{\max}\} = \pm\sqrt{2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}}$$

and

$$\frac{2K}{k_0} = \mathrm{Im}\{\Delta\kappa^{\max}\} = 2\sqrt{2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}}} \tag{27}$$

which represents the relationship for the coupling factor of contradirectional waves similar to (14).

Expression (26) can be transformed to the canonical form of the ellipse equation centered at $f = f_m$, which represents the normal form of the attenuation constant behavior in the vicinity of the MCP

$$\frac{(\mathrm{Im}\{\kappa\})^2}{a^2} + \frac{(f - f_m)^2}{b^2} = 1 \tag{28}$$

with

$$a^2 = 2\,\frac{H(\kappa_m, f_m)}{H''_{\kappa\kappa}} \quad b^2 = \frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}.$$

Other types of critical points, called fold or turning points [24], have been associated with the transition from a pair of real improper–improper solutions to a pair of complex conjugate (leaky mode) solutions [22]. If the complex mode occurs in the mode-coupling region (case $H(\kappa_m, f_m)/H''_{\kappa\kappa} > 0$), the coordinates of a fold point (κ_f, f_f) can be expressed in terms of functional characteristics about the MCP (κ_m, f_m). The obtained formulas (25) for the propagation constants $\kappa_{1,2}$ at boundary frequencies in condition (24) represent the approximation of the coordinates of the propagation constant at the fold points

$$\kappa_{f\,1,2} \approx \kappa_m \mp \frac{H''_{\kappa f}}{H''_{\kappa\kappa}}\sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}. \tag{29}$$

The frequency coordinates of the fold points can be approximated by the boundary frequencies in the condition (24)

$$f_{f\,1,2} \approx f_m \pm \sqrt{\frac{2H''_{\kappa\kappa}H(\kappa_m, f_m)}{(H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff}}}. \tag{30}$$

As a result, determination of the MCP's coordinates gives complete information about complex modes appearing in the mode-coupling region. The frequency range of complex mode regimes and coordinates of fold points are determined in terms of the MCP's characteristics.

III. Numerical Results and Discussion

To illustrate the concepts detailed above, examples of mode coupling on several different guided-wave structures will be considered in the following. In each case, a rigorous full-wave solution has been obtained. In the first example, a full-wave analysis of printed-circuit transmission lines has been performed using an electric-field integral-equation technique similar to [22] and [25]. A coupled set of homogeneous integral equations has been obtained, enforcing the boundary condition for the tangential components of the electric field on the surface of conducting strips. As another example, dispersion behavior of guided surface-wave modes on grounded slabs with anisotropic chirality has been studied via the volume equivalence theorem for bianisotropic media [26].

$$\jmath\,\mathrm{Im}\{\kappa_{1,2}\} = \pm\jmath\,\frac{\sqrt{2H''_{\kappa\kappa}H(\kappa_m, f_m) - ((H''_{\kappa f})^2 - H''_{\kappa\kappa}H''_{ff})(f - f_m)^2}}{H''_{\kappa\kappa}} \tag{26}$$

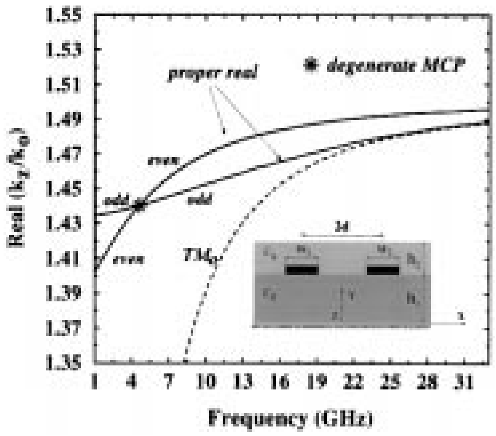

Fig. 1. Degeneracy of dominant EH_0 even and odd proper modes in a symmetrical conductor-backed coplanar strip line with infinite superstrate. The degenerate MCP is obtained at the point of intersection of even and odd dispersion curves: $w_1/h_1 = w_2/h_1 = 0.25$, $d/h_1 = 0.25$, $h_1 = 1$ cm, $h_2 = 0.1h_1$, $\epsilon_r = 2.25$, $\epsilon_s/\epsilon_r = 1.15$.

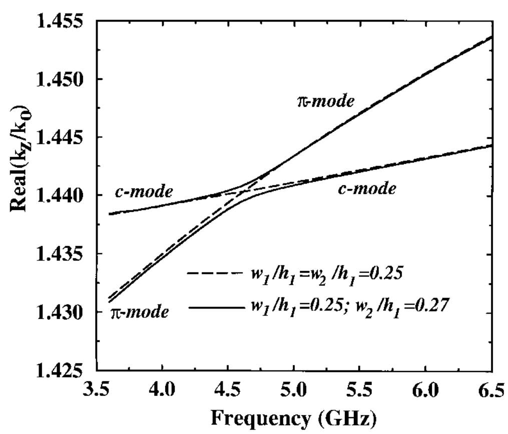

Fig. 2. Dispersion behavior for dominant modes of symmetrical and nonsymmetrical conductor-backed coplanar strips with infinite superstrate (see insert in Fig. 1). Degeneracy occurs for the symmetric structure (dashed line) and is broken by the perturbation of symmetry due to unequal strip widths (solid line): $d/h_1 = 0.25$, $h_1 = 1$ cm, $h_2 = 0.1h_1$, $\epsilon_r = 2.25$, $\epsilon_s/\epsilon_r = 1.15$.

The third structure is a shielded microstrip-like transmission line investigated using the method of integral equations for overlapping regions [27]. This method leads to a coupled system of an electric-field Fredholm-type integral equation of the second kind with a compact kernel.

The above three examples demonstrate codirectional coupling on three different physical structures. In the last example, contradirectional coupling is examined for improper modes on a printed transmission line exhibiting a complex-mode regime between the range of real solutions. In all cases examined here, a Galerkin's moment-method solution is applied to convert the system of integral equations into a matrix system (1).

Full-wave results for odd and even dominant proper forward modes in symmetrical conductor-backed coplanar strip line with infinite superstrate are demonstrated in Fig. 1. The ratio of dielectric permittivities of superstrate and substrate ϵ_s/ϵ_r is chosen about a critical value for which the propagation constant of the odd-mode approaches the TM_0 surface-wave mode, but does not become leaky. It is observed that a

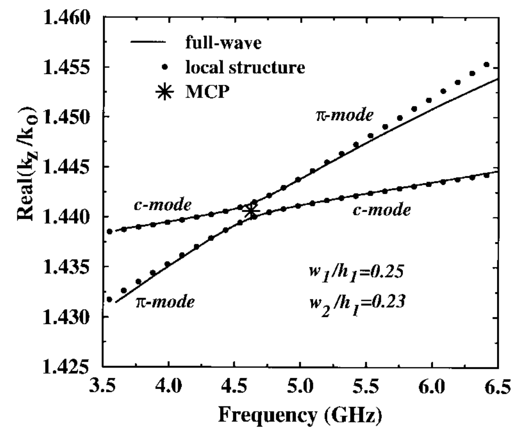

Fig. 3. Full-wave results and local structure for dominant modes of nonsymmetrical conductor-backed coplanar strips with infinite superstrate: $d/h_1 = 0.25$, $h_1 = 1$ cm, $h_2 = 0.1h_1$, $\epsilon_r = 2.25$, $\epsilon_s/\epsilon_r = 1.15$.

degeneracy of the dominant modes occurs at the frequency 4.6133 GHz, meaning that the mode-coupling factor at this point is equal to zero. The degenerate MCP [$H(\kappa_m, f_m) = 0$ as well as conditions in (2)] with coordinates $(\kappa_m, f_m) = (1.4403, 4.6133)$ is found at the point of the intersection of even and odd dispersion curves. The local structure generated in the vicinity of the MCP is represented by two intersected straight lines and agrees well with the full-wave solution. Equalization of even- and odd-mode velocities at the degeneracy point can be observed by varying a spectral or structural parameter of the symmetrical structure $(w_1 = w_2)$. Changes in width strip of one conductor $(w_1 \neq w_2)$ perturb the symmetry of the structure, leading to the transformation of odd and even modes into c- and π-modes, and form the mode-coupling region of codirectional waves shown in Fig. 2. It is observed (see Fig. 2) that small changes in width strip w_2/h_1 from 0.25 to 0.27 result in the formation of hyperbolic behavior of the dispersion curves of c- and π-modes. It should be noted that the appearance of c- and π-modes in nonsymmetrical structures is the result of coupling between odd and even modes of a symmetrical transmission line, and K/k_0 is a coupling factor between odd and even modes. The nondegenerate MCP with coordinates $(\kappa_m, f_m) = (1.4403, 4.6375)$ is obtained in the mode-coupling region, but is not shown in Fig. 2.

A decrease in strip width w_2/h_1 from 0.25 to 0.23 also leads to coupling of codirectional dominant modes $(H(\kappa_m, f_m)/H''_{\kappa\kappa} < 0)$. Fig. 3 demonstrates full-wave results and local structure for nonsymmetrical conductor-backed coplanar strips with infinite superstrate. Local structure (20) is generated in the vicinity of the MCP having coordinates $(\kappa_m, f_m) = (1.4406, 4.6247)$, as shown in Fig. 3. Very good agreement with full-wave dispersion characteristics behavior is observed in the mode-coupling region. The minimum of the coupling factor at $f = f_m$, defined by (13), is obtained in terms of functional characteristics about the MCP and equals 0.000 729.

Minimum coupling factor $(-2H(\kappa_m, f_m)/H''_{\kappa\kappa})^{1/2}$ behavior versus strip width w_2/h_1 is presented in Fig. 4 for the above discussed example. Changes in the coupling factor value are related to the migration of the MCP. A sharp minimum corresponding to zero coupling is obtained at the degeneracy

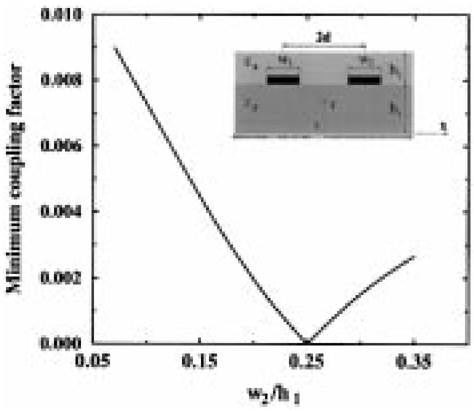

Fig. 4. Minimum coupling factor behavior versus w_2/h_1 for nonsymmetrical conductor-backed coplanar strip line with infinite superstrate: $w_1/h_1 = 0.25$, $d/h_1 = 0.25$, $h_1 = 1$ cm, $h_2 = 0.1h_1$, $\epsilon_r = 2.25$, $\epsilon_s/\epsilon_r = 1.15$.

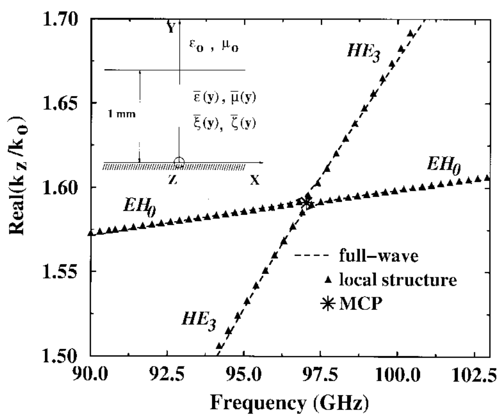

Fig. 5. Dispersion characteristics and local structure for EH_0 and HE_3 modes of an anisotropic chiral waveguide slab: $\epsilon_{xx} = 9\epsilon_0$, $\epsilon_{yy} = \epsilon_{zz} = 3\epsilon_0$, $\mu_{xx} = \mu_{yy} = \mu_{zz} = \mu_0$, $\xi_{\alpha\beta} = -\zeta_{\alpha\beta} = j\kappa_{\alpha\beta}\sqrt{\epsilon_0\mu_0}$, $\kappa_{xx} = 0.02$, $\kappa_{yy} = \kappa_{zz} = 0$. Achiral modes ($TM_0$, TE_3, not shown) intersect at a degenerate MCP. The addition of chirality κ_{xx} perturbs the symmetry of the structure, inducing hyperbolic mode-coupling behavior and a nondegenerate MCP.

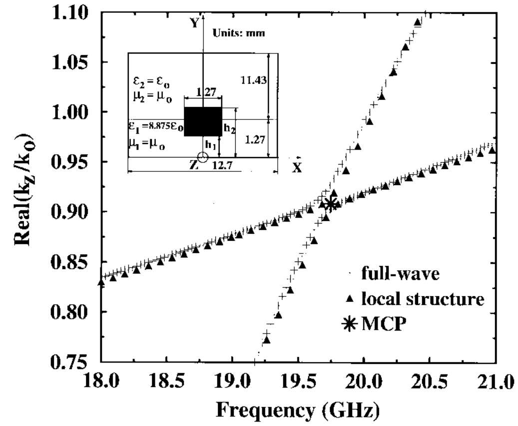

Fig. 6. Dispersion characteristics and local structure for E_z-odd–H_z-even higher order modes in semiburied microstrip line: $h_1 = 0.1$ mm, $h_2 = 1.2701$ mm.

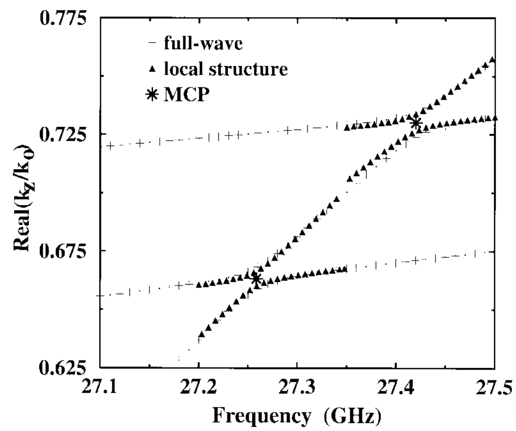

Fig. 7. Dispersion characteristics and local structure for E_z-even–H_z-odd higher order modes in semiburied microstrip line: $h_1 = 0.635$ mm, $h_2 = 1.271$ mm.

point (double-point bifurcation) of even and odd modes when $w_2/h_1 = 0.25$ (symmetrical structure). Slight changes in strip width (decrease or increase with respect to w_1/h_1) break a double-point bifurcation and form stable hyperbolic-type behavior of dispersion curves in the vicinity of the MCP. It should be noted that degeneracy of the MCP leads to instability of the structure with respect to width strip at the double point. Stability of the structure is predicted by nondegenerate MCP's obtained in the mode-coupling region of a nonsymmetrical transmission line.

As a second example, consider the anisotropic chiral waveguide slab shown in the insert of Fig. 5. Dispersion characteristics of hybrid surface-wave modes are obtained from the numerical solution of a system of coupled integral equations which are formulated using the volume equivalence principle for bianisotropic media. Fig. 5 shows dispersion curves in the mode-coupling region for the normalized propagation constant k_z/k_0 and the local structure generated about the MCP. The MCP with coordinates $(\kappa_m, f_m) = (1.5912, 97.0352)$ is obtained as the solution of a system of nonlinear equations (2). Chirality κ_{xx} acts as a parameter of structural perturbation, leading to coupling of independent TE and TM achiral modes of an unperturbed anisotropic waveguide slab. The degeneracy of achiral modes is removed with the addition of κ_{xx}, resulting in formation of the dispersion curves' hyperbolic behavior in the vicinity of the MCP. The local structure in the mode-coupling region of chiral modes is stable, which is guaranteed by the presence of the nondegenerate MCP in the local vicinity. Instability of the structure occurs when chirality parameter κ_{xx} goes to zero, leading to degeneration of achiral modes. Although not shown here, a similar perturbation can be achieved in the absence of chirality by rotation of the optical axis away from the geometrical axes of the waveguiding structure [29].

The third example of codirectional coupling arises in the dispersion characteristics for E_z-odd–H_z-even and E_z-even–H_z-odd higher order modes of a shielded microstrip-like transmission line partially buried in the substrate, as shown in Figs. 6 and 7. The geometry of the structure is depicted in the insert of Fig. 6. Local structures are obtained using representation (20) for mode coupling of codirectional waves. MCP's are determined at the minimum of coupling between longitudinal-section electric (LSE) and longitudinal-section magnetic (LSM) background modes of the layered waveguide. MCP found in the coupling region of odd modes has coordinates $(\kappa_m, f_m) = (0.9079, 19.7411)$ (see Fig. 6).

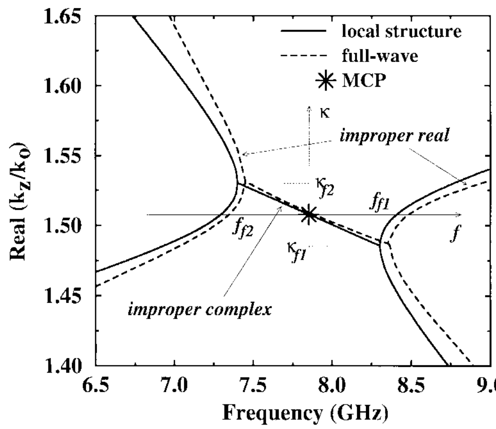

Fig. 8. Mode coupling of contradirectional improper modes for conductor-backed coplanar strips in case of $w_1/h_1 = w_2/h_1 = 0.372$. Improper complex mode occurs in the mode-coupling region. Local structure and functional characteristics are obtained in the vicinity of the MCP: $d/h_1 = 0.311$, $h_1 = 1$ cm, $\epsilon_r = 2.25$, $\epsilon_s = 1$.

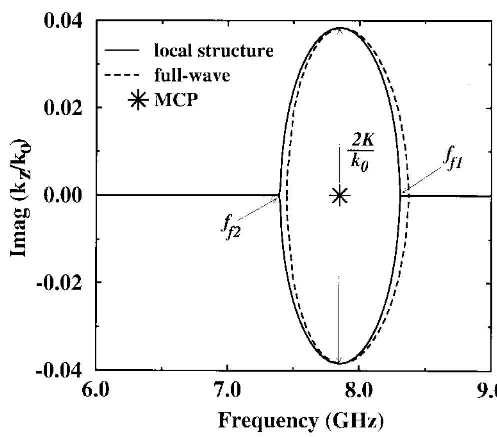

Fig. 9. Full-wave results and local structure for attenuation constants versus frequency for $w_1/h_1 = w_2/h_1 = 0.372$.

Fig. 7 shows MCP's determined in coupling regions of even modes with coordinates $(\kappa_{m1}, f_{m1}) = (0.6627, 27.2586)$ and $(\kappa_{m2}, f_{m2}) = (0.7297, 27.4193)$, respectively. The presence of the strip results in a perturbation of the structure, leading to coupling of background modes and formation of hyperbolic-type behavior of hybrid modes' dispersion characteristics in mode-coupling regions.

As a final example, the spectrum of all possible solutions, including proper and improper real and complex modes, has been investigated for a symmetrical conductor-backed coplanar strips geometry [22], [28], shown in the insert of Fig. 1. It has been observed that small changes in strip width $w/h_1 \, (w_1 = w_2 = w)$ from 0.370 to 0.372 can significantly change qualitative behavior of improper real solutions and generate an improper complex nonphysical mode [22], [28]. Note that the behavior of improper real and improper complex modes in the region of interest is considered in this example for illustrative purposes, showing the connection with the contradirectional coupling formalism from the coupled-mode theory. The similar characteristic behavior in mode-coupling regions and the occurrence of complex modes in those regions have been observed in various guided-wave structures, including shielded nonreciprocal finline [23], lossless shielded boxed microstrips [30], [31], and diffraction gratings [18], which may be explained using the proposed approach based on the concept of MCP's.

Fig. 8 shows the propagation constants behavior for strip width $w/h_1 = 0.372$. The local structure (4) satisfying the complex mode condition (22) is represented by a straight-line equation (23) for $\mathrm{Re}\{\kappa(f)\}$ and canonical form of the ellipse equation centered at $f = f_m$ (28) for $\mathrm{Im}\{\kappa(f)\}$, as shown in Figs. 8 and 9. The coupling factor K/k_0, defined by relationship (27) at Morse frequency f_m for contradirectional waves, is equal to 0.038 35. Two branches of a hyperbola are obtained out of the frequency range (24) generating local structure (4) about the MCP with coordinates $(\kappa_m, f_m) = (1.5079, 7.8516)$. It is found that coordinates of fold points as transition points from a pair of improper real solutions to a complex conjugate improper mode can be approximated by functional characteristics obtained in the vicinity of MCP's. Coordinates of fold points (κ_{f1}, f_{f1}) and (κ_{f2}, f_{f2}), depicted in Fig. 8, have been calculated using approximation formulas (29) and (30), and found as (1.4855, 8.3039) and (1.5303, 7.3992) respectively. Coordinates of fold points presented in [22] are (1.4865, 8.3688) and (1.5315, 7.4473), which have been obtained using definition of those points and the full-wave solution. Note that the dispersion characteristics shown in Figs. 8 and 9 represent general behavior of natural modes in the region of contradirectional coupling for a variety of waveguiding structures mentioned above.

IV. Conclusion

The concept of MCP's from catastrophe theory is presented to show the relationship with the traditional coupled-mode formalism. The expression for the coupling factor is obtained in terms of functional properties of the characteristic determinant in the vicinity of the MCP, related to the minimum of mode coupling. Mode coupling between codirectional and contradirectional waves is considered and investigated using analytically obtained formulas for local structures about MCP's. The complex mode appearing in the mode-coupling regime of contradirectional waves is explained, and complex mode frequency range is determined. Approximate formulas for coordinates of fold points are obtained using the local representation of the function in the vicinity of the MCP. A geometrical interpretation of mode coupling and complex-mode regimes is demonstrated by simple normal forms showing qualitative dispersion behavior in the regions of interest. Numerical results, including the full-wave solution and generated local structures about MCP's, are obtained for various types of guided-wave open and shielded structures. It is shown that the presence of nondegenerate MCP's in mode-coupling regions is related to structural stability in the local vicinity. Degeneracy of MCP's leads to instability of the structure with respect to certain perturbations and formation of a double-point bifurcation. The concept of MCP's in conjunction with a full-wave solution can be effectively used for identification, analysis, and explanation of interesting effects associated with mode coupling.

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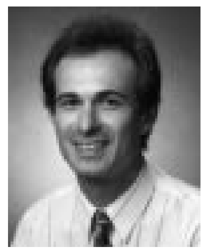

Alexander B. Yakovlev (S'94–M'97) was born on June 5, 1964, in the Ukraine. He received the Ph.D. degree in radiophysics from the Institute of Radiophysics and Electronics, National Academy of Sciences, Ukraine, in 1992, and the Ph.D. degree in electrical engineering from the University of Wisconsin–Milwaukee, in 1997.

From 1992 to 1994, he was an Assistant Professor in the Department of Radiophysics, Dniepropetrovsk State University. From 1994 to 1997, he was a Research and Teaching Assistant in the Department of Electrical Engineering, University of Wisconsin–Milwaukee, and a Post-Doctoral Fellow in the Electrical and Computer Engineering Department, North Carolina State University, Raleigh. From 1997 to 1998, he worked as an R&D Engineer with Ansoft Corporation, Compact Software Division, Pittsburgh, PA. He is currently a Research Associate in the Department of Electrical Engineering, University of Wisconsin–Milwaukee. His current research interests include electromagnetic modeling of printed-circuit transmission lines, theory of leaky waves, and catastrophe and bifurcation theories.

Dr. Yakovlev received the 1992 Young Scientist Award presented at the URSI International Symposium on Electromagnetic Theory, Sydney, Australia, and the 1996 Young Scientist Award presented at the International Symposium on Antennas and Propagation, Chiba, Japan.

George Hanson (S'85–M'91) was born in Glen Ridge, NJ, in 1963. He received the B.S.E.E. degree from Lehigh University, Bethlehem, PA, in 1986, the M.S.E.E. degree from Southern Methodist University, Dallas, TX, in 1988, and the Ph.D. degree from Michigan State University, East Lansing, in 1991.

From 1986 to 1988, he was a Development Engineer with General Dynamics, Fort Worth, TX, where he worked on radar simulators. From 1988 to 1991, he was a Research and Teaching Assistant in the Department of Electrical Engineering, Michigan State University. He is currently Associate Professor of electrical engineering and computer science at the University of Wisconsin–Milwaukee. His research interests include electromagnetic-wave phenomena in layered media and microwave characterization of materials.

Dr. Hanson is a member of URSI Commission B, Sigma Xi, and Eta Kappa Nu.